Physics of Nuclear Magnetism Laboratory course
Assignment 1

Frequency Tuning and Impedance Matching of NMR Tank Circuits

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Abstract

A successful nuclear magnetic resonance (NMR) experiment hinges on three main skill sets needed to be mastered by every student new to this field: knowledge of the electronics of NMR probes and spectrometer, spin excitation using radio frequency (RF) pulses and detection of the faint nuclear precession of excited spin systems. Typically, electronics and detection are well controlled by commercially available NMR systems. While this might be advantageous for standard experiments, it severely limits the capabilities of NMR experiments which are "out-of-the-ordinary". Thus, in this first assignment we will learn how to set up an NMR tank circuit, conduct frequency tuning at desired resonance frequencies while maintaining optimal power transfer and how to characterise radio frequency circuits.

1 Theoretical background

For these experiments, you will need the following theoretical background. Please prepare necessary mathematical expressions and derivations:

- What is an LC tank (or resonance) circuit? Describe the energy transformation within this circuit in one oscillation. (equilibration of capacitive and inductve energies)
- Derive the expression for the resonance frequency of a LC tank circuit. For that, solve the Thomson equation:
$$E_{total}(t) = \frac{L}{2}I(t)^2 + \frac{1}{2C}Q(t)^2 \tag{1}$$
with E_{total} the total stored energy in a LC circuit, L and C denote the inductance and capacitance, and $Q(t)$ as well as $I(t) = \dot{Q}(t)$ the time dependent electric charge and current respectively. Hint: $E_{total}(t)$ is a conservative physical quantity, i.e. $\dot{E}_{total}(t) = 0$.
The resulting second order linear ordinary differential equation can easily be solved using the Ansatz:
$$Q(t) = \hat{Q} sin(\omega t + \phi) \tag{2}$$
with $\hat{Q}$ the amplitude of (2), $\omega = 2\pi f$ the circular frequency and ϕ the phase of the oscillation.
- Derive the analytical expression for the inductance of a long solenoidal coil (3) and compare it to the empirical formula usually used in electronics (4):
$$L_{analytical} = \frac{\mu_0 N_{turns}^2 A_{coil}}{l_{coil}} \tag{3}$$
$$L_{empirical}[\mu H] = \frac{0.394 r_{coil}^2[cm^2] N_{coil}^2}{9r_{coil}[cm] + 10 l_{coil}[cm]} \tag{4}$$
How much different is the result for a microcoil ($l_{coil} = 150\mu m, r_{coil} = 75\mu m, N_{turns} = 6$) when L is calculated by (4) compared to the analytical value for long solendoids (3)?

- What is the skin effect in a conductor? Using the microscopic Ohms law, show that the following expression is a good approximation of the ac resistance of a solenoidal coil at radiowave frequencies ($\sim$ 100 MHz). Assume the coils length be given by $l_{coil} \approx 2\pi r_{coil} N_{coil}$ (r_{coil} is the diameter of the coil and N_{coil} is the number of turns)

$$R \approx \sqrt{\pi f \rho \mu} \cdot \frac{r_{coil} N_{coil}}{r_{wire}} \tag{5}$$

  with r_{wire} the conductor radius and r_{coil} the radius of the solenoid.

- What is the quality factor Q of a resonance circuit and how is it defined? Estimate Q of an LC circuit with a solenoidal coil of 10 turns, 0.5 mm copper wire thickness and a diameter r_{coil} of 3 mm operating at 300 MHz. Skin effects are not negligible at these frequencies.

- What are frequency tuning and impedance matching?

- Briefly review magnetic susceptibility and permeability.

2 Tasks

Please work on the tasks step-by-step and summarize your observations thoroughly and logically when you had in the assignment.

1. using 750 μm copper wire, prepare the following coil geometries:

   - coil A: 1 mm diameter, 4 turns
   - coil B: 3 mm diameter, 8 turns
   - coil C: 5 mm diameter, 12 turns

   Using a multimeter, read out the dc-resistance of each coil. Connect one coil to the NMR probe interface in the correct manner.

2. Determine the minimal and maximal resonance frequency (pending optimal impedance matching) of each coil using the spectrum analyser.

3. Determine the quality factors of each coil at these maximal/minimal resonance frequencies. Do the quality factors solely scale with frequency, if not, why? Do the quality factors of each coil at the maximal/minimal resonance frequencies match the theoretical value for Q? What could be the reasons for non-ideality?

4. Connect **coil B** to the probe. Tune and match to the approximate mid point of its frequency range. Now, insert sample 1 (rubber) in the coil and note the change in resonance frequency observed on the spectrum analyser. Repeat with sample 2 (mysterious sample with a metallic luster). What do you observe? Why do the frequencies change when the samples were inserted? What could be the nature of sample 2? Try to estimate their magnetic susceptibility from the observed frequency shifts on the spectrum analyser.

3 Literature

- Eiichi Fukushima, *Experimental pulse NMR a nuts and bolts approach*

- Dustin Wheeler and Mark Conradi, *Practical Exercises for Learning to Construct NMR/MRI Probe Circuits*

- Noel Mispelter, *NMR probeheads for biophysical and Biomedical Experiments*

All items in the list can be found in the shared data server of HPSTAR